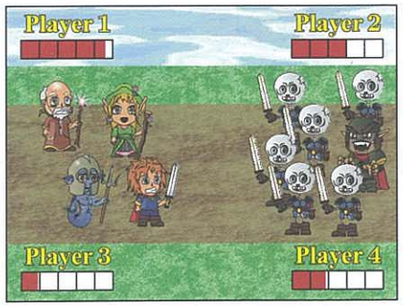
3 In a 4 player video game, the red bars show how healthy each player is.
Which player has a health level of:

a 60% **b** 36%
c 91% **d** 17%?

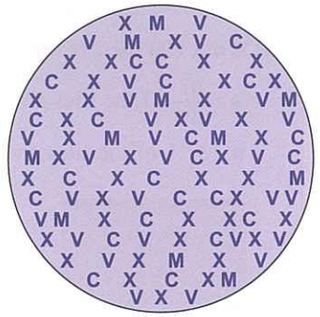


4 There are 100 symbols in the circle.

a What fraction of the symbols are:
i M **ii** C **iii** X **iv** V?

b What percentage of the symbols are:
i M **ii** C **iii** X **iv** V?

c Find the sum of the percentages in **b**. Explain this result.

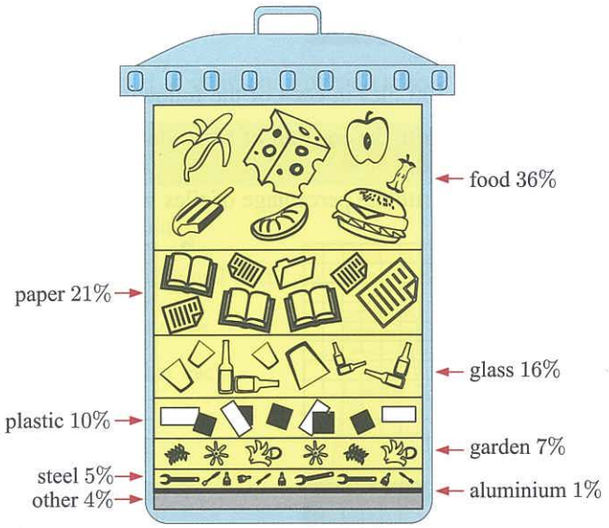


5 The diagram illustrates the different things which we throw away.

a Check that the percentages add to 100%.

b Does paper or glass make up the greater percentage of waste?

c What percentage of waste is:
i food **ii** steel
iii either plastic or aluminium?



B CONVERTING BETWEEN PERCENTAGES AND FRACTIONS

CONVERTING PERCENTAGES TO FRACTIONS

To convert a percentage into a fraction, we first write the percentage as a fraction with denominator 100. We then express the fraction in simplest form.

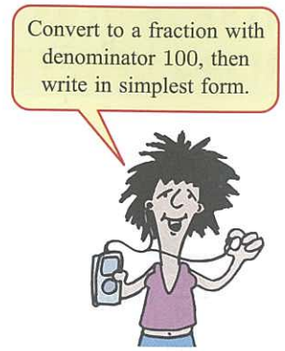
Example 2 — Self Tutor

Write as a fraction:

a 29% **b** 85%

a
$$29\% = \frac{29}{100}$$

b
$$85\% = \frac{85}{100} = \frac{85 \div 5}{100 \div 5} = \frac{17}{20}$$



EXERCISE 10B.1

1 Write as a fraction:

a 59% **b** 13% **c** 3% **d** 97%

2 Write as a fraction in simplest form:

a 10% **b** 50% **c** 90% **d** 5%
e 22% **f** 74% **g** 15% **h** 65%
i 25% **j** 80% **k** 35% **l** 75%
m 4% **n** 48% **o** 56% **p** 64%

CONVERTING FRACTIONS TO PERCENTAGES

Many fractions can be converted to percentage form by first writing the fraction with denominator 100.

Example 3 — Self Tutor

Write as a percentage:

a $\frac{19}{100}$ **b** $\frac{2}{5}$ **c** $\frac{557}{1000}$

a
$$\frac{19}{100} = 19\%$$

b
$$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\%$$

c
$$\frac{557}{1000} = \frac{557 \div 10}{1000 \div 10} = \frac{55.7}{100} = 55.7\%$$

EXERCISE 10B.2

1 Write as a percentage:

a $\frac{21}{100}$ **b** $\frac{53}{100}$ **c** $\frac{91}{100}$ **d** $\frac{8}{100}$
e $\frac{3}{10}$ **f** $\frac{7}{10}$ **g** $\frac{0}{10}$ **h** $\frac{10}{10}$

2 Write as a percentage:

a $\frac{1}{2}$ **b** $\frac{13}{50}$ **c** $\frac{1}{5}$ **d** $\frac{41}{50}$ **e** $\frac{3}{20}$

f $\frac{3}{5}$ **g** $\frac{7}{25}$ **h** $\frac{19}{20}$ **i** $\frac{12}{25}$ **j** $\frac{19}{25}$

3 Write as a percentage:

a $\frac{29}{200}$ **b** $\frac{231}{1000}$ **c** $\frac{759}{1000}$ **d** $\frac{103}{500}$

4 **a** Write $\frac{200}{100}$ in simplest form. **b** Write 2 as a percentage.

5 Use the illustration to complete the table below.

PRINTABLE TABLE

	Students	*Number*	*Fraction*	*Fraction with denominator 100*	*Percentage*
a	wearing shorts				
b	with a ball				
c	not wearing a hat				
d	wearing shorts and with a ball				
e	wearing track pants, baseball cap, and a green top				
f	wearing shorts or long pants				
g	wearing shoes				

6 Copy and complete these patterns:

a $\frac{1}{5} = 20\%$
$\frac{2}{5} = ...$
$\frac{3}{5} = ...$
$\frac{4}{5} = ...$
$\frac{5}{5} = ...$

b $\frac{1}{4}$ is 25%
$\frac{2}{4}$ is .....
$\frac{3}{4}$ is .....
$\frac{4}{4}$ is .....

c $\frac{1}{3}$ is $33\frac{1}{3}\%$
$\frac{2}{3}$ is .....
$\frac{3}{3}$ is .....

d 1 is 100%
$\frac{1}{2}$ is 50%
$\frac{1}{4}$ is .....
$\frac{1}{8}$ is .....
$\frac{1}{16}$ is .....

C CONVERTING BETWEEN PERCENTAGES AND DECIMALS

CONVERTING PERCENTAGES TO DECIMALS

To write a percentage as a decimal number, we **divide by 100%**.

Since $100\% = \frac{100}{100} = 1$, dividing by 100% is the same as dividing by 1. We therefore do not change the value of the number.

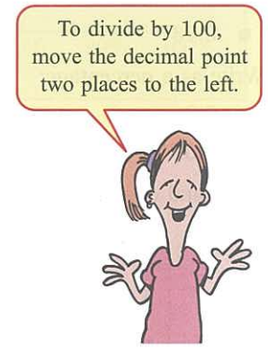
Example 4 — Self Tutor

Write as a decimal:

a 21% **b** 6.7%

a 21%
$= 21. \div 100$
$= 0.21$

b 6.7%
$= 06.7 \div 100$
$= 0.067$



EXERCISE 10C.1

1 Write as a decimal:

a 10% **b** 50% **c** 25% **d** 5%

e 33% **f** 57% **g** 94% **h** 6%

i 40% **j** 11% **k** 1% **l** 90%

2 Write as a decimal:

a 17.5% **b** 81.6% **c** 60.7% **d** 9.4%

e 3.9% **f** 4.3% **g** 1.7% **h** 0.8%

3 Write each percentage as:

i a decimal **ii** a fraction in simplest form.

a 71% **b** 65% **c** 30% **d** 8%

CONVERTING DECIMALS TO PERCENTAGES

To write a decimal number as a percentage, we **multiply by 100%**.

Example 5 — Self Tutor

Write as a percentage:

a 0.27 **b** 0.055

a $0.27 = 0.27 \times 100\% = 27\%$

b $0.055 = 0.055 \times 100\% = 5.5\%$

EXERCISE 10C.2

1 Write as a percentage:

a 0.37 **b** 0.89 **c** 0.15 **d** 0.49

e 0.73 **f** 0.11 **g** 0.05 **h** 0.02

2 Write as a percentage:

a 0.2 **b** 0.7 **c** 0.9 **d** 0.4

e 0.074 **f** 0.739 **g** 0.086 **h** 0.001

3 Copy and complete:

	Percent	Fraction	Decimal
a	20%		0.2
b	40%	$\frac{2}{5}$	
c			0.5
d		$\frac{3}{4}$	
e			0.85

	Percent	Fraction	Decimal
f		$\frac{2}{25}$	
g			0.35
h	84%		
i	100%		
j		$\frac{3}{20}$	

4 Write:

a 28% as a fraction and as a decimal **b** $\frac{4}{5}$ as a percentage and as a decimal

c 0.45 as a percentage and as a fraction **d** 0.25 as a percentage and as a fraction.

DISCUSSION

- Does it make sense to talk about percentages greater than 100%, such as 110% or 250%?
- What would be meant by the statement "My business has grown by 300% over the last 2 years."?

D NUMBER LINES

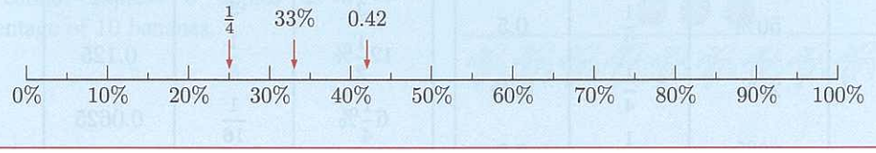
Plotting numbers on a number line can be difficult when the numbers are given as a mixture of fractions, decimals, and percentages. However, we can make the process easier by converting all fractions and decimals to percentages.

Example 6 — Self Tutor

Convert $\frac{1}{4}$, 0.42, and 33% to percentages, and place them on a number line.

- $\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 25\%$
- $0.42 = 0.42 \times 100\% = 42\%$
- 33% is already a percentage

We use the percentages to place the numbers on a number line.



EXERCISE 10D

1 Convert each set of numbers to percentages, and place them on a number line:

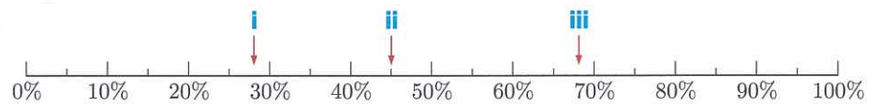
a $\frac{3}{5}$, 70%, 0.65 **b** 55%, $\frac{9}{20}$, 0.83 **c** 0.93, 79%, $\frac{17}{20}$

d 0.85, $\frac{3}{4}$, 92% **e** $\frac{27}{50}$, 67%, 0.59 **f** 47%, 0.74, $\frac{7}{10}$

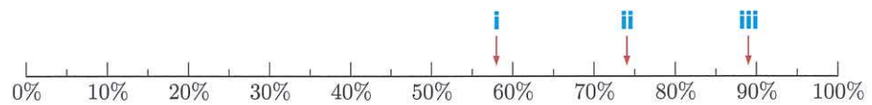
g $\frac{3}{4}$, 0.65, 42% **h** 0.39, 58%, $\frac{7}{20}$, $\frac{2}{5}$ **i** $\frac{661}{1000}$, 73%, $\frac{13}{20}$, 0.47

2 Write each of the following number line positions as a fraction with denominator 100, as a decimal, and as a percentage:

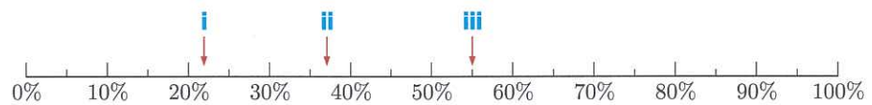
a



b



c



3 **a** Write each fraction or decimal as a percentage:

i $\frac{17}{25}$ **ii** 0.43 **iii** $\frac{39}{50}$ **iv** 0.627

b Place the values in **a** on a number line.

c Hence write the values in order from smallest to largest.

COMMON CONVERSIONS

Some percentages occur frequently in business calculations and advertisements, so it is useful to learn their values as fractions.

Copy this table of common conversions, and add others if you wish:

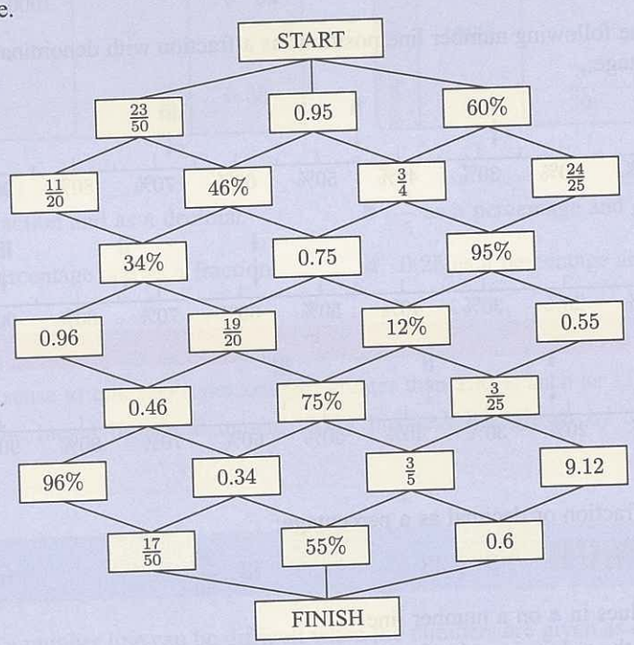
Percentage	*Common fraction*	*Decimal*
100%	1	1.0
75%	$\frac{3}{4}$	0.75
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
10%	$\frac{1}{10}$	0.1
5%	$\frac{1}{20}$	0.05

Percentage	*Common fraction*	*Decimal*
$33\frac{1}{3}\%$	$\frac{1}{3}$	0.3333 ....
$66\frac{2}{3}\%$	$\frac{2}{3}$	0.6666 ....
$12\frac{1}{2}\%$	$\frac{1}{8}$	0.125
$6\frac{1}{4}\%$	$\frac{1}{16}$	0.0625
$\frac{1}{2}\%$	$\frac{1}{200}$	0.005

PUZZLE

In this Puzzle, you must find a path from the Start to the Finish so that no two numbers on the path have the same value.

PRINTABLE PUZZLE



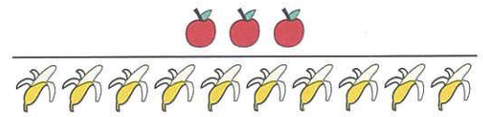
E ONE QUANTITY AS A PERCENTAGE OF ANOTHER

Percentages are often used to compare quantities, so it is useful to express one quantity as a percentage of another.

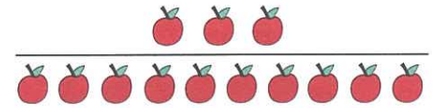
We must be careful to only compare **like with like**.

For example:

We cannot express 3 apples as a percentage of 10 bananas.



However, we *can* express 3 apples as a percentage of 10 apples.



We must also make sure that the quantities are compared in the same units.

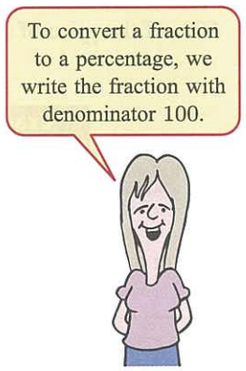
For example, if we are asked to express 35 cm as a percentage of 7 m, we would first convert the larger unit to the smaller one. In this case we would find 35 cm as a percentage of 700 cm.

To express one quantity as a percentage of another, we first write them as a fraction, and then convert the fraction to a percentage.

Example 7 Self Tutor

Express a mark of 22 out of 25 as a percentage.

$$\frac{22 \text{ marks}}{25 \text{ marks}} = \frac{22}{25}$$
$$= \frac{22 \times 4}{25 \times 4}$$
$$= \frac{88}{100}$$
$$= 88\%$$



EXERCISE 10E

1 Express as a percentage:

a 17 marks out of 20
b 11 marks out of 25
c 37 marks out of 50
d 138 marks out of 200

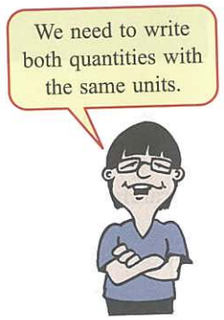
Example 8 Self Tutor

Express the first quantity as a percentage of the second:

a 60 cm, 3 m **b** 160 g, 2 kg

a
$$\frac{60\text{ cm}}{3\text{ m}} = \frac{60\text{ cm}}{300\text{ cm}} = \frac{60 \div 3}{300 \div 3} = \frac{20}{100} = 20\%$$

b
$$\frac{160\text{ g}}{2\text{ kg}} = \frac{160\text{ g}}{2000\text{ g}} = \frac{160 \div 20}{2000 \div 20} = \frac{8}{100} = 8\%$$



2 Express the first quantity as a percentage of the second:

a 20 cm, 100 cm **b** 10 km, 50 km **c** 3 m, 4 m
d 7 mm, 2 cm **e** 50 g, 1 kg **f** 84 cm, 4 m
g 48 seconds, 5 minutes **h** 720 kg, 2 tonnes **i** 63 cents, 9 dollars
j 24 minutes, 10 hours **k** 50 cents, $25 **l** 1 mL, 1 litre

3 Express as a percentage:

a 72 diners in a restaurant that seats 200 diners
b 405 books sold out of 500 printed
c 660 square metres of lawn in a 2000 square metre garden
d 28 000 spectators in a 40 000 seat stadium
e a ten pin bowler scores 186 points out of a possible 300 points
f €60 off the price of a television marked at €400.

Example 9 Self Tutor

In a class of 25 students, 6 have black hair.
What percentage of the class have black hair?

The fraction with black hair $= \frac{6}{25} = \frac{6 \times 4}{25 \times 4} = \frac{24}{100}$

So, 24% of the class have black hair.

4 In a class of 25 students, 13 have blue eyes. What percentage of the class have blue eyes?

5 2 km of gas pipes need to be laid. So far, 480 m of pipes have been laid. What percentage of the pipes have been laid?

6 There are 50 singers in a school choir. 12 of them are in Grade 4, 17 are in Grade 5, and 21 are in Grade 6.

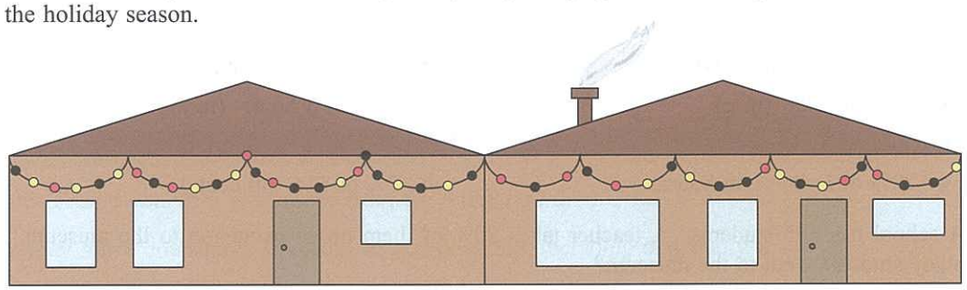
a Find the percentage of singers who are in:
i Grade 4 **ii** Grade 5 **iii** Grade 6.

b Check that the sum of the percentages in **a** is 100%.

7 Maria is taking a cooking course with 20 classes. To earn her certificate, she needs to attend at least 80% of the classes. Maria was unable to attend 3 classes.

a What percentage of the classes did Maria attend?
b Will Maria receive her certificate?

8 The Smith family and the Jones family each put up a display of coloured lights on their house during the holiday season.



Smith *Jones*

a Find the percentage of lights which are working on each house.
b Which house has the higher percentage of working lights?
c Of all the lights that are working, what percentage are:
i red **ii** yellow?

F FINDING PERCENTAGES OF QUANTITIES

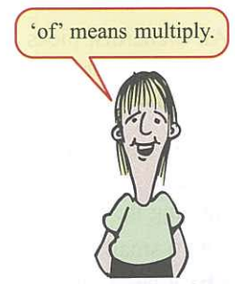
To find a percentage of a quantity, we first convert the percentage to a decimal. We then multiply to find the answer.

Example 10 Self Tutor

Find 53% of 4000 people.

53% of 4000 people
$= 0.53 \times 4000$ people $\{53\% = 0.53\}$
$= 2120$ people

Calculator: 0.53 × 4000 =



EXERCISE 10F

1 Find:

a 15% of $200 **b** 80% of 250 people **c** 27% of 30 kg

d 75% of 320 litres **e** 7% of 70 cm **f** 45% of 35 seconds

Example 11 — Self Tutor

Find 12% of 3 km, giving your answer in m.

12% of 3 km
$= 0.12 \times 3000$ m $\quad \{12\% = 0.12, \quad 3 \text{ km} = 3000 \text{ m}\}$
$= 360$ m $\quad$ *Calculator*: 0.12 [×] 3000 [=]

2 Find the following, giving each answer in the units indicated:

a 27% of $1 (in cents) **b** 5% of 9 m (in cm)

c 35% of 2 kg (in g) **d** 10% of 3 hours (in min)

e 60% of 8 kL (in L) **f** 42% of 4 cm (in mm)

g 22% of 5 days (in hours) **h** 7% of $14 (in cents)

3 A school has 485 students. A teacher takes 20% of them on an excursion to the museum. How many students went to the museum?

4 A council collects 4500 tonnes of rubbish each year. 27% of the rubbish is recycled. How many tonnes of rubbish is this?

5 15% of an energy drink is sugar. How many grams of sugar are there in a 450 g can of energy drink?

6

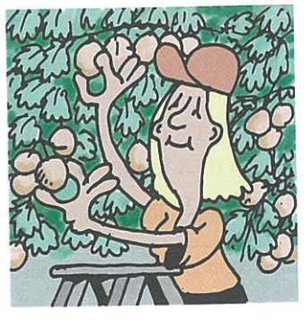
30% of a farmer's crop was barley, and the rest was wheat.

a What percentage of the crop was wheat?

b If the farmer planted 2400 acres in total, how many acres were planted with:

i barley **ii** wheat?

7 An orchardist picks 2400 kg of apricots for drying. 85% of the weight is lost in the drying process. How many kilograms of dried apricots are produced?



8 A new company policy requires 5% of the workers in each office to have first aid training. How many workers need first aid training in:

a a small office containing 20 workers

b a large office containing 300 workers?

9 In a series of triathlon races, prize money is awarded to the top three competitors. The winner receives 50% of the prize money, second place receives 35%, and third place receives 15%.

The total prize money and results for the first two races are shown alongside.

Find the prize money won so far by:

a Shane **b** Daniel.

	Total prize money	1st	2nd	3rd
Race 1	$3000	Daniel	Matt	Shane
Race 2	$5000	Justin	Daniel	Trent

10 When a painting is sold at an art gallery, the art gallery receives a fixed percentage of the selling price, and the artist receives the rest.

a When a painting was sold for $200, the art gallery received $66.

i What percentage of the price did the art gallery receive?

ii What percentage of the price did the artist receive?

b A second painting is sold for $450. How much money will be received by the:

i art gallery **ii** artist?

PUZZLE

In this Puzzle your task is to move from the START to the FINISH.

You may move one square at a time, in any direction, including diagonally.

However, you may only move to a square if the row percentage of the column number is a whole number.

For example, you could move from the start to the top left square, since 50% of 40 is 20, which is a whole number.

PRINTABLE GRID

Column numbers

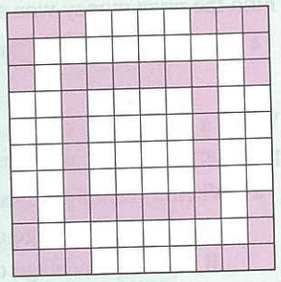
Row percentages	40	55	25	24	70	50	65	60
	START							
50%								
25%								
8%								
40%								
$33\frac{1}{3}\%$								
75%								
150%								
5%								
	FINISH!							

KEY WORDS USED IN THIS CHAPTER

- decimal
- fraction
- percentage

REVIEW SET 10A

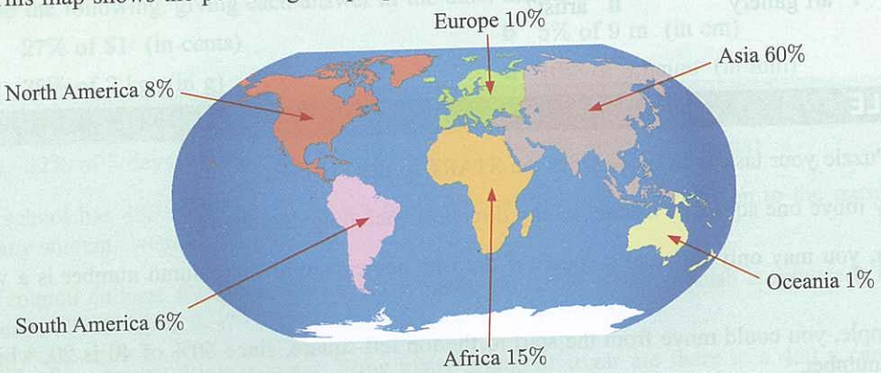
1 There are 100 tiles in the pattern shown.



a Write the fraction of tiles which are coloured, leaving your answer with denominator 100.

b Write the percentage of tiles which are coloured.

2 This map shows the percentage of people living in each continent.



a Find the sum of the percentages.

b What percentage of the world's population lives in:

i Asia **ii** North or South America?

c What *fraction* of the world's population lives in:

i Europe **ii** Africa?

3 Write as a percentage: **a** 0.47 **b** 0.306

4 Write as a fraction in simplest form:

a 31% **b** 16% **c** 94%

5 Find:

a 45% of £60 **b** 12% of 4 m (in cm)

6 50 students attended the annual quiz night. 27 of the students won at least one prize during the night. What percentage of the students won at least one prize?

7 Write $\frac{221}{1000}$ as a percentage.

8 Write as a decimal:

a 81% **b** 2% **c** 10.8%

9 Marcia has travelled 620 kilometres of a 2000 km journey. What percentage of the journey has she travelled?

10 Write $\frac{3}{4}$, 0.78, and 72% as percentages, and then place them on a number line.

11 A small country town has 280 households. 45% use a wood burning fire to warm their homes, 30% use electricity, 15% use gas, and 10% use oil or kerosene. How many households use:

a electricity **b** fire or gas?

12 Brenda makes her own mayonnaise. 40% of the mayonnaise is egg, 35% is oil, 20% is lemon juice, and the rest is mustard.

a Does the mayonnaise contain more oil or lemon juice?

b What percentage of the mayonnaise is mustard?

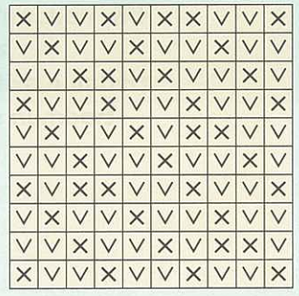
c How much:

i lemon juice is there in 300 mL of mayonnaise

ii oil is there in 800 mL of mayonnaise?

REVIEW SET 10B

1 There are 100 symbols in the pattern shown.



a Count the number of each type of symbol.

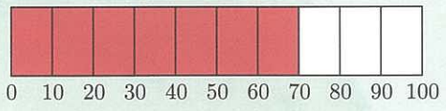
b Write the number of each type as a fraction of the total.

c Write the number of each type as a percentage of the total.

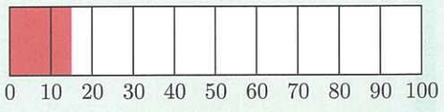
d Check that your percentages in **c** sum to 100%.

2 Estimate the percentage shaded in each diagram:

a



b



3 Write as a percentage:

a 0.09 **b** 0.136 **c** 0.702

4 In a group of 200 children, 34 are allergic to peanuts. What percentage of the children are allergic to peanuts?

5 Write 74% as a fraction and as a decimal.

6 8% of the students at a school are left-handed. The school has 375 students. How many students at the school are left-handed?

7 Write as a percentage:

a $\frac{27}{100}$ **b** $\frac{18}{25}$ **c** $\frac{13}{20}$

8 Express the first quantity as a percentage of the second:

a 13 goals from 25 shots **b** 58 cm of 2 m

9 Klaus spent €15 from the €50 he was given for his birthday. What percentage of his money did he spend?

10 A cordial mixture contains 15% cordial and 85% water. How much:

a cordial is in a 200 mL glass of cordial mixture

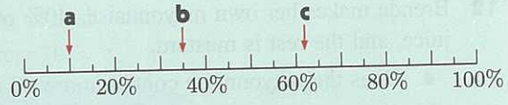
b water is in an 800 mL bottle of cordial mixture?

11 Write each number line position indicated as:

i a fraction with denominator 100

ii a decimal

iii a percentage.



12 600 people applied to become a firefighter, but only 24 of them were accepted into the training program. What percentage of the people who applied were accepted?

Chapter 11

Positive and negative numbers

Contents: